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A TIME QUASI-CRYSTAL

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The great attention recently devoted to quasi-crystals has popularised the method of showing quasi-periodicity by the cut and projection of a regular lattice, plotting each of the periodicities on a separate axis. The date of Easter can be seen in this light.

The periods of the Sun and Moon are incommensurable, being respectively 365.2422 and 29.53059 (synodic month) mean solar days. Meton of Athens in 432 BC found that 235 months are closely equal to 19 Solar years, and this is thus a rational convergent, good to about 13 parts per million.

Incommensurability has led to various methods of calculating the date of Easter, which is the Christian festival of Resurrection, superimposed on the primordial festival of the rebirth of plant and animal life around the Spring Equinox. The bitter feud between the Eastern and Western branches of the Christian Church led to the schism at the Council of Nicea in 325 AD on (inter alia) this issue. Indeed, we may trace the enduring split between Eastern and Western Europe to incommensurability.

The present legal definition used in Britain is that "Easter-Day is the first Sunday after the first full Moon which happens upon, or next after, the 21st. day of March (the vernal equinox) and if the full Moon happens upon a Sunday, Easter-Day is the Sunday after. This definition is contained in an Act of Parliament (24 Geo. II., cap. 23)".[4] The Moon referred to is not the real Moon but the Paschal Moon, which is a theoretical construct, running a day or two behind. "This was done purposely, to avoid the chance of concurring with the Jewish Passover, which the framers of the calendar seem to have considered a greater evil than celebrating Easter a week too late." The matter is of immense complexity as may be seen from the article "Calendar", in the 11th Edition (1911) of the Encyclopaedia Britannica, from which the quotation comes.

Tables of the date of Easter show an irregular time sequence, the earliest date being 21 March and the latest 23 April. However if the date is plotted on a second axis, perpendicular to the year axis, a regular two-dimensional lattice is obtained (Fig. 1).

The most usual circumstance is that each year Easter is 8 days later. The length of the year is 365.2422 mean solar days. Thus, since $365 = 1 \mod 7$, corres-

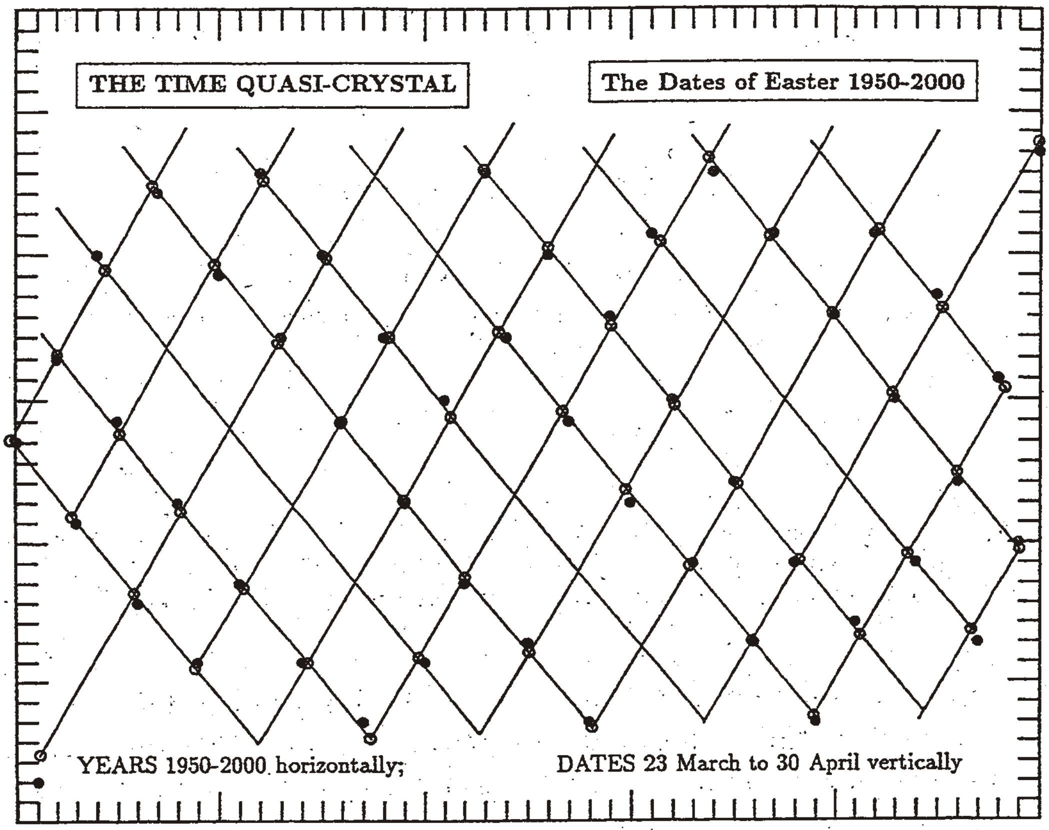

Fig. 1. The celestial pineapple or the time quasi-crystal. A plot of the date of Easter against the year, from 1950 to 2000 (filled points), with a superimposed lattice (open circles).

ponding Sundays are one day later in the year. The length of the synodic month is 29.5306 mean solar days so that the full Moon is either 10.63 days earlier or 18.89 days later. If it is taken as 11 days earlier the date of Easter is a week earlier; this, plus the drop back of one day of the week, gives the usual advance of 8 days per year. If Easter is later the usual drop back is 21 or 14 days (a whole number of weeks) less the one day (for 365 = 1 mod 7) making 20 or 13 days. The occurrence of Leap Year (366 = 2 mod 7) causes a drop back of 2 days making a change of 19 or 12 days (later) or 9 days (earlier).

We get almost a regular lattice because the lattice point ($H = 4$, $K = 3$, below) corresponds closely to a shift of 28 days.

The lattice has been fitted to the observed points by a least squares procedure, the x and y coordinates (respectively the years and the dates) being fitted separately, with three parameters each.

The resulting equations of the lattice points are

$4.07\,H + 3.81\,K + 2.31$ = date in April (thus March dates are negative);

$2.35\,H - 3.05\,K + 1972.013$ = year; where H and K are integers.

The dates and years are the nearest integers to the results given, except for 1951 which is a case when the following Sunday rule, mentioned above, applies.

Thus, as in the case of quasi-crystals,[2] graphic presentation clarifies apparent irregularity through the use of a higher dimensional space.

A quasi-crystal has two incommensurable periods in the same direction and, following the work of Kramer and Neri,[1] can be seen to be a three-dimensional projection of a six-dimensional lattice. This method of representation is due originally to de Wolff,[3] who added a fourth dimension, perpendicular to the others, to show the incommensurate modulation of an ordinary three-dimensional crystal. In mathematics a quasi-periodic function is one which is the sum of a finite number of incommensurably periodic functions.

My interest is in the extraction of intelligence or regularities from apparently unstructured information. Similar results might be extracted from observations of the tides and the referee has kindly checked from an almanack that a table of the Full Moon can be treated in the same way. Without wishing to press the analogy too far, the disturbing effects of the incidence of Leap Year might be considered as phasons.

References

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3. P. M. De Wolff, *Acta Crystallogr.* **A30** (1974) 777–785.
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A.L. Mackay "A time quasi-crystal" Mod. Phys. Let. B4, 989-991, (1990).

Easter Is a Quasicrystal

Ian Stewart reveals the divine mathematics of a holiday

Ten years ago my first Mathematical Recreations column was about Fermat's Christmas Theorem. With the Lenten season upon us, it seems only fitting to devote this 96th, and my final, column to Easter.

Christmas always falls on December 25, but Easter is quite another matter. The holiday can fall on any date between March 22 and April 25, a five-week window. The date changes from year to year for a number of reasons. First, the date has to be a Sunday because the crucifixion occurred on a Friday and the resurrection on a Sunday. Second, the New Testament says the crucifixion took place during the Jewish holiday of Passover, which is celebrated for eight days following the first full moon of spring.

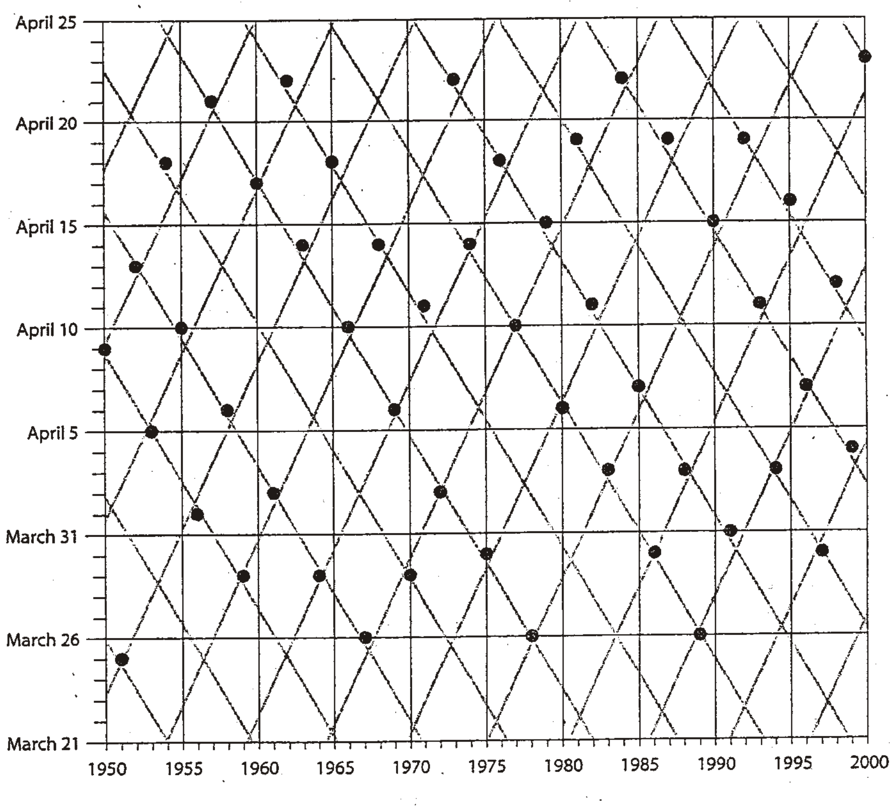
The date of Easter is thus linked to several astronomical cycles, and it is here that the difficulties arise. The lunar month is currently about 29.53 days long, and the solar year about 365.24 days long. This leads to 12.37 lunar months per year, an inconvenient relationship because it is not an integer. It so happens that 235 lunar months are very close to 19 solar years, and the church's system for assigning a date to Easter exploits this coincidence. In A.D. 325 at the Council of Nicaea, church leaders decided that Easter should fall on the first Sunday after the first full moon occurring on, or after, the spring equinox (the date in March on which day and night have equal length).

The year was based on the Julian calendar back then, with one leap year in every four. The dates of full moons were assumed to repeat every 19 Julian years; a bit of juggling with the calendar made this period equal to 235 lunar months. The 19-year period was called the lunar cycle, and each year's position in this cycle was indicated by its Golden Number, which ran from 1 to 19. The entire cycle of the Julian calendar would repeat every 76 years—after four lunar cycles of 19 years each, the pattern of leap years would repeat. The mathematical principle here is that the length of a cycle is equal to the lowest common multiple of the lengths of its constituent subcycles (76 is the lowest common multiple of 19 and 4).

Easter dates repeated in a 532-year cycle, because 532 is the lowest common multiple of 76 (the Julian calendar's cycle) and 7 (the cycle of days in the week). It was a tidy system, but unfortunately it did not accurately account for the true lengths of the lunar month and solar year. As the centuries passed, the calendar started to slip relative to the seasons. (Dante, the medieval Italian poet, pointed out that eventually January would cease to be part of winter.) Finally, in 1582 Pope Gregory XIII reformed the calendar by omitting the leap day in all years that are multiples of 100, except for years that are multiples of 400 (such as the year 2000, for example). To correct for the previous slippage, the Gregorian calendar for 1582 skipped the 10 days between October 4 and 15.

The church's new procedure for calculating the date of Easter assigned each year a number called the Epact, an integer between 0 and 29. This number reveals the phase of the moon on January 1 of each year, with 0 indicating a new moon and 15 a full moon. Every so often the cycle of Epacts must be revised to account for the fact that 235 lunar months do not exactly equal 19 solar years. The last such correction occurred in 1900, and the next will be needed in 2200.

Using the Golden Number and Epact for any given year, one can calculate the date of the first full moon of spring. To determine the date of Easter—the first Sunday



EASTER QUASICRYSTAL is created by graphing the dates of the holiday for the years 1950 to 2000. The graph resembles a regular lattice (*green*), although the plot points (*orange*) vary slightly from the lattice intersections, like the arrangement of atoms in a quasicrystal.

ALL ILLUSTRATIONS BY BRYAN CHRISTIE

Calculating Easter

Choose any year of the Gregorian calendar and call it x. To determine the date of Easter, carry out the following 10 calculations (it's easy to program them on a computer).

1. Divide x by 19 to get a quotient (which we ignore) and a remainder A. This is the year's position in the 19-year lunar cycle. ($A + 1$ is the year's Golden Number.)
2. Divide x by 100 to get a quotient B and a remainder C.
3. Divide B by 4 to get a quotient D and a remainder E.
4. Divide $8B + 13$ by 25 to get a quotient G and a remainder (which we ignore).
5. Divide $19A + B - D - G + 15$ by 30 to get a quotient (which we ignore) and a remainder H. (The year's Epact is $23 - H$ when H is less than 24 and $53 - H$ otherwise.)
6. Divide $A + 11H$ by 319 to get a quotient M and a remainder (which we ignore).
7. Divide C by 4 to get a quotient J and a remainder K.
8. Divide $2E + 2J - K - H + M + 32$ by 7 to get a quotient (which we ignore) and a remainder L.
9. Divide $H - M + L + 90$ by 25 to get a quotient N and a remainder (which we ignore).
10. Divide $H - M + L + N + 19$ by 32 to get a quotient (which we ignore) and a remainder P.

Easter Sunday is the Pth day of the Nth month (N can be either 3 for March or 4 for April). The year's dominical letter can be found by dividing $2E + 2J - K$ by 7 and taking the remainder (a remainder of 0 is equivalent to the letter A, 1 is equivalent to B, and so on).

Let's try this method for $x = 2001$: (1) $A = 6$, (2) $B = 20$, $C = 1$, (3) $D = 5$, $E = 0$, (4) $G = 6$, (5) $H = 18$, (6) $M = 0$, (7) $J = 0$, $K = 1$, (8) $L = 6$, (9) $N = 4$, (10) $P = 15$. So Easter 2001 is April 15.

after the full moon—the church assigned each year a dominical letter, from A to G, indicating the date of that year's first Sunday: A for January 1, B for January 2, and so on. Every leap year has two dominical letters, one for January and February and the other for the remaining months.

The system has its flaws. The church considers March 21 to be the perennial date of the vernal equinox, but the real astronomical equinox can occur as early as March 19 (as will happen in 2096). Also, the moon does not slavishly follow ecclesiastical conventions. In 1845 and 1923 the first full moon of spring occurred after Easter Sunday in the world's easterly longitudes.

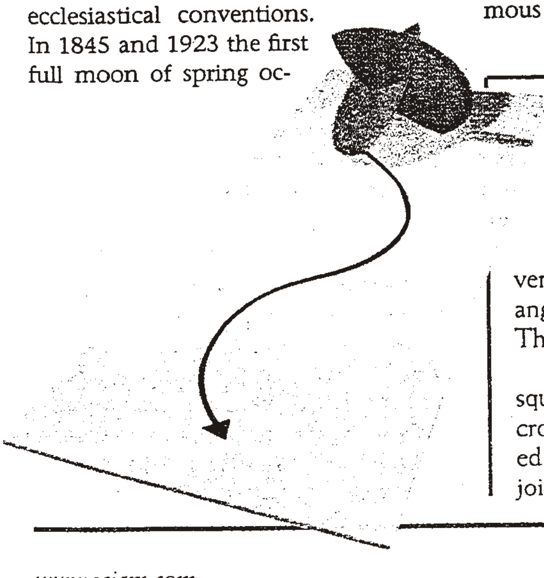
In 1800 German mathematician Carl Friedrich Gauss invented a simple algorithm that incorporated the church's rules for calculating Easter's date. Unfortunately, Gauss's work contained a minor oversight: it gives April 13 for the year 4200 when the correct date should be April 20. He corrected this error by hand in his own copy of the published paper. The first flawless algorithm was presented in 1876 in the journal *Nature* by an anonymous American. In 1965 Thomas H. O'Beirne of Glasgow University published two such procedures in his book *Puzzles and Paradoxes* (Oxford University Press). O'Beirne's method puts the various cycles and adjustments into an arithmetical scheme [*see box at left*].

In general terms, the date of Easter slips back by about eight days each year until it hops forward again. The pattern looks irregular but actually follows the arithmetical procedure just described. In 1990 Alan Mackay, a crystallographer at the University of London, realized that this near-regular slippage ought to show up in a graph that compared the date of Easter with the number of the year [*see illustration on opposite page*]. The result is approximately a regular lattice, like the arrangement of atoms in a crystal.

The peculiarities of the calendar, however, make the dates vary slightly as compared with the lattice. The graph more closely resembles a quasicrystal, a molecular structure built for the first time in the early 1980s. Quasicrystals are not as regular as crystals, but their arrangement of atoms is by no means random. The structure is similar to a curious class of tilings discovered by University of Oxford physicist Roger Penrose; these tilings cover the plane without repeating the same pattern periodically. The atoms of quasicrystals have the same near regularity, as do the dates of Easter. The holiday is a quasicrystal in time rather than space.

Under the rules of the Gregorian calendar, the cycle of Easter dates repeats exactly after 5,700,000 years. Long before the first repeat, though, the rules will have slipped relative to astronomical realities: the lengths of the month and day are slowly changing, mainly because of tidal friction. Just for the fun of it, though, try to calculate Easter's date for the year 1000000. ■

(ANSWER: April 16)



READER_FEEDBACK

Readers continue to comment on the solid known as the sphericon ["Cone with a Twist," October 1999]. John D. Determan of Alhambra, Calif., and Cecil Deisch of Warrenville, Ill., suggested using a cone with a 60-degree vertex. When sliced in half, this figure has a cross section that is an equilateral triangle, and two such half-cones can be glued together with a 120-degree twist. The resulting object rolls, but not very far, unfortunately.

David Racusen of Shelburne, Vt., suggested starting with a cylinder with a square cross section and joining two halves with a 90-degree twist. And Don Bancroft of Brookfield, Ill., sent me a copy of his U.S. patent (number 4,257,605, dated March 24, 1981) describing a rolling device made from two semicircles that are joined at the middle of their straight sides with a 90-degree twist (*left*). —*I.S.*